Final Exam Calculus 2 (EN)

Problem 1 (15 points)

Show that the series $\sum_{n=0}^{\infty} (2^n + 3^n + 4^n)/5^n$ is convergent (10 points)

Give the value of the series (5 points)

Problem 2 (20 points) Consider the series $\sum_{n=1}^{\infty} \frac{(x-7)^n}{n\,(3^n)}$

(10 points) For which x is the series absolute convergent; (5 points) For which x is conditionally convergent ?; (5 points) For which x is divergent?

Problem 3 (15 points)

a) (7 points) Show that the distance D of a point P(x1,y1,z1) from the surface

$2x+3y+4z+D_1=0$ is: $D = | 2x1 + 3y1 + 4z1 + D_1 | / \sqrt{29}$

b) (8 points) Show that the distance D_{1-2} between the two surfaces

$2x+3y+4z+D_1=0$ en $2x+3y+4z+D_2=0$ is: $D_{1-2} = | D_1 - D_2 | / \sqrt{29}$

Problem 4 (20 points)

Consider the function $f(x,y) = xy^4/(x^2+y^8)$. Does the limit $\lim_{(x,y)\to(0,0)} f(x,y)$ exist?

Problem 5 (20 points)

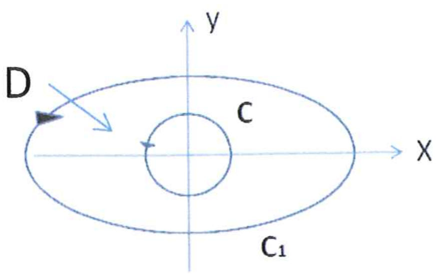
a) Change the integration order for calculate the integral (15 points) $\int_{x=0}^{x=4}\int_{y=x/2}^{y=2}(4-y^2)^{1/4}\,dy\,dx$; and then

b) Calculate the volume $\iiint dv$ between the surfaces z=25 and $z = x^2 + y^2$ (5 points)

Problem 6 (15 points) a) (5 points) Consider the vector field $\vec{F} = P(x)\vec{i} + Q(y)\hat{j}$

Show that for every closed path C: $\oint_C \vec{F}\cdot d\vec{r} = 0$

b) (10 points) Consider the vector field $\vec{F} = P(x,y)\vec{i} + Q(x,y)\hat{j}$ with $\partial Q/\partial x = \partial P/\partial y$ in D



C: is a circle around the origin with: $\oint_C \vec{F}\cdot d\vec{r} = S$

Calculate the integral: $\oint_{C_1} \vec{F}\cdot d\vec{r} = ?$